Name: ______________________________

Confidence Interval Quiz Review

Your quiz tomorrow will be VERY much like this review! So FINISH IT!!! YOU KNOW WHO YOU ARE! :)

1. A manufacturer of flashlights wants to know how well one of their newer styles is selling in a chain of large home-improvement stores. They select a simple random sample of 20 stores, record how many of the flashlights were sold in a 30-day period, and construct a 95% confidence interval for the mean number of flashlights sold.

   (a) If, instead of constructing a 95% confidence interval, the flashlight manufacturer constructed a 98% confidence interval, would the 98% interval be wider, narrower, or the same width as the 95% interval? Explain.

   (b) How would the width of confidence interval change if the flashlight manufacturer took a larger sample? Explain.

   (c) The 20 stores in the sample were actually the only stores who provided sales figures from 36 stores that were randomly chosen to be in the sample. Can the manufacturer adjust the confidence interval to take this nonresponse into account? If so, how? If not, why not?

2. A university health services physician is concerned about how much sleep freshman are getting in the first few months of school. She asks a simple random sample of 20 students how much sleep they got the previous night and constructs a 95% confidence interval for the mean amount of sleep in hours.

   (a) If, instead of constructing a 95% confidence interval, the physician constructed a 90% confidence interval, would the 90% interval be wider, narrower, or the same width as the 95% interval? Explain.

   (b) How would the width of confidence interval change if the physician took a larger sample? Explain.

   (c) After calculating the interval, the physician realizes that the sample was drawn only from the 70% of freshman who had turned in their health forms by the time they arrived on campus. Can she adjust the confidence interval to take this undercoverage into account? If so, how? If not, why not?

3. Suppose you know that the distribution of finishing times for a certain crossword puzzle has a mean of 25 minutes, a standard deviation of 8 minutes, and is moderately skewed left. You take an SRS of 45 finish times from this distribution and calculate the mean finish time, $\overline{x}$.

(a) Describe the shape, center, and spread of the sampling distribution of $\overline{x}$.

(b) Find a number, k, such that 95% of the values in the sampling distribution will lie within k minutes of the mean of the distribution.

(c) If you take repeated samples of size 45 from this population, what proportion of the time will the interval $\overline{x} \pm k$ contain the number 25? Explain.

4. An insect ecologist reports a 95% confidence interval for the mean length of full-grown aquatic larvae of the Phantom Midge *Chaoborus albatus* to be 6.9 to 8.5 mm, based on a sample of 9 individual larvae.

   (a) What are the point estimate and margin of error associated with this confidence interval?

   (b) If the ecologist had reported a 99% confidence interval instead of a 95% interval, how would it have been different? Explain.

   (c) The ecologist was unhappy with how wide this interval was. What should he do to produce a narrower interval with the same level of confidence? Explain.

5. A recent poll found that "433 of the 1548 randomly-selected U.S. adults questioned felt that unemployment compensation should be extended an additional six months while the country is in its current economic downturn." We want to use this information to construct a 95% confidence interval to estimate the proportion of the U.S. adults who feel this way.

   (a) State the parameter our confidence interval will estimate.

   (b) Identify the conditions that must be met to use this procedure, and explain how you know that each one has been satisfied.

   (c) Find the appropriate critical value and the standard error of the sample proportion.

   (d) Give the 95% confidence interval.

   (e) Interpret the confidence interval constructed in part D. in the context of the problem.

   (f) Suppose you wanted to estimate the proportion of people who feel that unemployment compensation should be expanded with 95% confidence to within ± 1.5%. Calculate how large a sample you would need.

(g) If you wanted to have a margin of error of ±1.5% with 99% confidence, would your sample have to be larger, smaller, or the same size as the sample in part F.? Explain.

(h) This poll was conducted by randomly calling cell phone numbers. Explain how undercoverage could lead to a biased estimate in this case, and speculate about the direction of bias.

6. A *New York Times* poll on women's issues interviewed 1025 women randomly selected from the United States, excluding Alaska and Hawaii. The poll found that 47% of the women said they do not get enough time for themselves.

   (a) Construct and interpret a 90% confidence interval that estimates the proportion of women in the United States who do not feel that they get enough time for themselves. Use the four-step process.

   (b) Explain, in the context of this problem, what "90% confidence" means.

   (c) Suppose this poll was conducted by telephone calls made from 9 am to 5 pm. Explain how using this method might result in biased results, and speculate about the direction of bias.

7. You want to conduct a poll at your school to estimate with 95% confidence the proportion of students in your school who have outside jobs in the evenings and on weekends. You'd like your margin of error to be less than ±5%.

   (a) How large must your sample be to produce a 95% confidence interval with the desired margin of error of ±5%?

   (b) How big does your school have to be for this interval to be accurate? Explain.

Confidence Interval Quiz Review
Answer Section

OTHER

1. ANS:
A. If our interval has to capture the true mean 98% of the time in repeated samples instead of only 95% of the time, it will have to be wider. B. If the sample size is larger, the standard deviation of the sampling distribution will be smaller, so the confidence interval will be narrower. C. It's not possible to adjust the confidence interval to compensate for bias inherent in the data collection methods. The margin of error for a confidence intervals only includes chance variation, not other sources of error like nonresponse.

PTS: 1

2. ANS:
A. If our interval has to capture the true mean only 90% of the time in repeated samples instead of 95% of the time, an narrower interval can be constructed. B. If the sample size is larger, the standard deviation of the sampling distribution will be smaller, so the confidence interval will be narrower. C. It's not possible to adjust the confidence interval to compensate for bias inherent in the data collection methods. The margin of error for a confidence intervals only includes chance variation, not other sources of error like undercoverage.

PTS: 1

3. ANS:
A. $\mu_{\bar{x}} = 25; \sigma_{\bar{x}} = \dfrac{8}{\sqrt{45}} \approx 1.193$. Since $n = 45$, the central limit theorem applies, so the shape of the sampling distribution is approximately Normal. B. From Table A, 95% of the scores in a Normal distribution are within ±1.96 standard deviations of the mean, so $k = 1.96 \cdot \dfrac{8}{\sqrt{45}} \approx 2.34$ minutes.
C. Since will be within ±k minutes of 25 minutes in 95% of samples, will contain 25 minutes in 95% of samples.

PTS: 1

4. ANS:
Point estimate = midpoint of interval = 7.7 mm. Margin of error = half of the width of the interval = 0.8 mm. B. The 99% confidence interval would have to be wider in order to contain the true mean length in 99% of repeated samples, instead of only 95% of samples. C. Take a larger sample, which will reduce the standard deviation of the sampling distribution and make the interval narrower.

PTS: 1

5. ANS:
A. The parameter is the true proportion of adults who feel that unemployment compensation should be extended an additional six months. B. Random: The problem states that the subjects were "randomly selected." The sample of 1548 is obviously less than 10% of the population. Large counts: $n\hat{p} = 433 \geq 10; n\left(1-\hat{p}\right) = 1115 \geq 10$. C. The critical value for 95% confidence, based on Table A, is $z^* = 1.96$, and since $\hat{p} = \dfrac{433}{1548} \approx 0.280$, the standard error is $\sqrt{\dfrac{(0.28)(0.72)}{1548}} \approx 0.0114$.
D. $0.280 \pm 1.96(0.0114) \longrightarrow 0.280 \pm 0.022 \longrightarrow \left(0.258, 0.302\right)$. E. We are 95% confident that the interval from 0.258 to 0.302 contains the true proportion of adults who feel that unemployment compensation should be extended an additional six months. F. Assuming p is close to our sample $\hat{p} = 0.28$, we want a sample size n such that $(1.96)\sqrt{\dfrac{(0.28)(0.72)}{n}} \leq 0.015$. Solving as an equality produces $n = 3442.07$, so n must be 3443 to ensure a margin of error below 0.015. G. Since a 99% confidence interval will have a critical z^* of 2.57 instead of 1.96, n will have to be larger to make the margin of error 0.015. H. A poll conducted by cell phone will miss individuals in the population who do not own cell phones. It's possible that a higher proportion of unemployed people do not have cell phones, so this would cause our sample to underestimate the proportion of adults who feel that unemployment compensation should be extended an additional six months.

PTS: 1

6. ANS:
A. State: We wish to estimate, with 90% confidence, the true proportion of women in the continental 48 states who feel that they don't get enough time for themselves. Plan: We will use a one-sample z-interval for a population proportion. Conditions: *Random*: The problem states that women were randomly selected, and the sample of 1025 is clearly less than 10% of women in the lower 48 states. *Large counts:* $n\hat{p} = (1025)(0.47) = 481.75 \geq 10; n\left(1-\hat{p}\right) = 543.25 \geq 10$ Do: 90% confidence interval is $0.47 \pm 1.645\left(\sqrt{\dfrac{(0.47)(0.53)}{1025}}\right) \longrightarrow 0.47 \pm 0.026 \longrightarrow \left(0.444, 0.496\right)$. Conclude (i.e., interpret the interval): We are 90% confident that the interval 0.444 to 0.496 contains the true proportion of women in the U.S. (excluding Alaska and Hawaii) who feel that they don't have enough time for themselves. B. If a large number of confidence intervals were constructed in this way, about 90% of them would contain the true proportion of women in the U.S. who feel that they don't have enough time for themselves. C. Women who work outside the home are unlikely to be home between 9 am and 5 pm, so they would be under-represented in the sample. If we assume that these women are busier than those who do not work outside the home—or at least are less likely to be alone at home for some part of the day—our sample will underestimate the proportion of women who feel they don't have enough time for themselves.

PTS: 1

7. ANS:
A. Using the conservative value $\hat{p} = 0.5$, we want a sample size n such that $(1.96)\sqrt{\frac{(0.5)(0.5)}{n}} \le 0.05$. Solving as an equality produces $n = 384.16$, so n must be 385 to ensure a margin of error below 0.05. B. To satisfy the 10% condition when sampling a finite population, the number of students must be at least 10x385 = 3,850. A big school!

PTS: 1